Power, Type I and Type II Errors – Topic 18

A baseball player has been a .250 career hitter works to improve his batting average. He wants to convince the team manager that he has improved, and the manager offers him a trial of 30 at-bats to convince him.

a. State the null and alternative hypotheses in symbols and in words.

p = prop. of hits the baseball player will make (actual batting avg.)

$H_0: p = .250$ He has not improved – stayed same

$H_a: p > .250$ He has improved

Decision:	H_0 is True	H_0 is False
Reject H_0	Type I Error	☆
Fail to Reject H_0	☆	Type II Error

b. Describe what a Type I error means in context.

Reject H_0 when it's true:

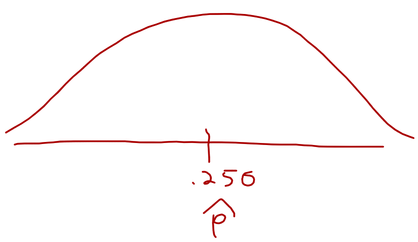
Have evid. that he improved, but he really stayed the same.

c. Describe what a Type II error means in context.

Fail to reject H_0, when it's false:

Not enough evid. to say he improved, but he really did.

d. Describe the sampling distribution for his proportion of hits out of 30 attempts, assuming the null hypothesis is true and he really has a .250 batting average.

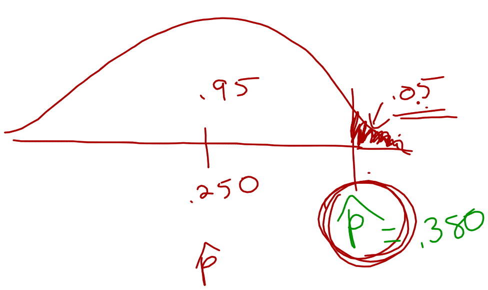


$n = 30$

$\sigma_{\hat{p}} = \sqrt{\frac{.250(1-.250)}{30}} = .0791$

e. Assuming he is still a .250 hitter, find what proportion of hits he would have to get to have a .05 or less chance of doing that well.

$\alpha = .05$



invnorm(.95) = 1.645 = z

$1.645 = \frac{\hat{p} - .250}{.0791}$

$\hat{p} = .380$

his goal → you will reject H_0 if his $\hat{p}$ is .380 or higher @ $\alpha = .05$

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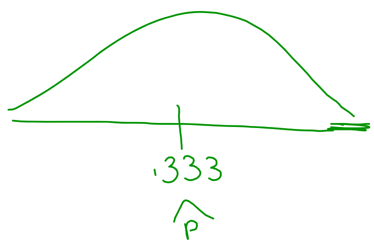
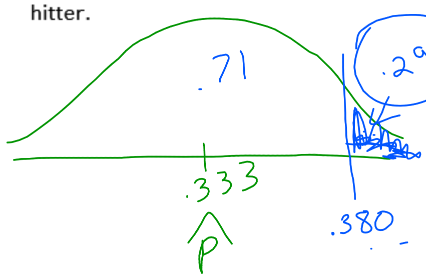
"new" p

f. Now let's assume that he really did improve (the null hypothesis is false) and will now have a .333 average. Describe the sampling distribution for 30 trials at this average.



$$\sigma_{\hat{p}} = \sqrt{\frac{.333(1-.333)}{30}} = .0860$$

g. Using your answer from e, find the probability that he hits at least that proportion if he is a .333 hitter.

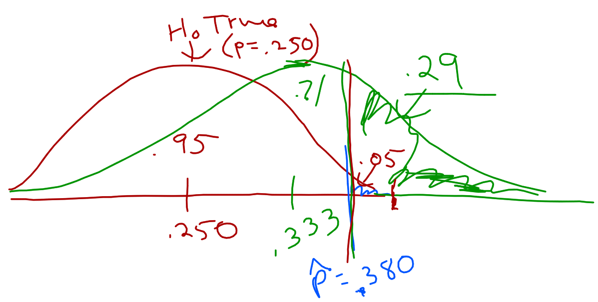


$$z = \frac{.380 - .333}{.0860} = .55$$

$$P(z > .55) = .29$$

h. Fill in the chart with the probabilities of each event.

Decision:	H_0 is True	H_0 is False
Reject H_0	Type I Error $\alpha = .05$	power = .29
Fail to Reject H_0	.95	Type II .71 $\beta = .71$



i. List the three factors that will affect the power of a test.

1. ↑ n → ↑ power (↓ Type II / Don't change Type I)
2. ↑ α → ↑ power (↓ Type II / ↑ Type I
3. "new" p (.333) was farther away from "old" p ($H_0: p = .250$) (↓ Type II / Don't change Type I)

↑ Type I / ↓ Type II

↓ Type I / ↑ Type II

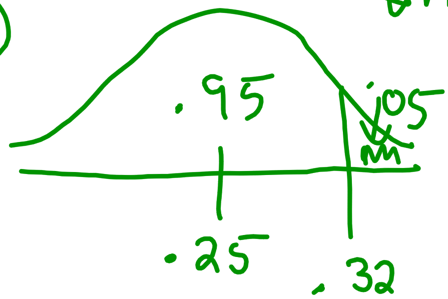
The next pages show the difference the changes in part i) and how the affect the Type I, Type II, and power calculations.

1) redoes the work on the notes worksheet (so use it as a reference), but changes the sample size from 30 to 100.

2) redoes the work on the notes, but changes the significance level from .05 to .10

The third change on the notes is if the 'new' p is farther away from the hypothesized p, the power would also increase. This would be if all of the front page of the notes stayed the same, but the new batting avg. of .333 from the back is increased even more to .400. You could redo the calculations for parts f and g, using .400 instead of .333. I didn't actually do this part on the board, but the power would increase again.

1



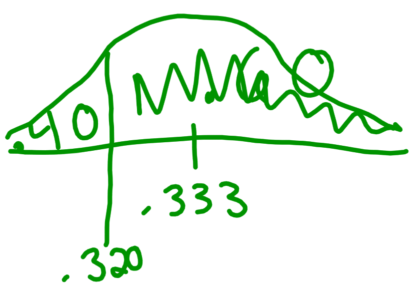
$n = 100$

$$\sigma_{\hat{p}} = \sqrt{\frac{.25(1-.25)}{100}}$$

$$\sigma_{\hat{p}} = .0433$$

$$1.645 = \frac{\hat{p} - .25}{.0433}$$

$$\hat{p} = .32$$



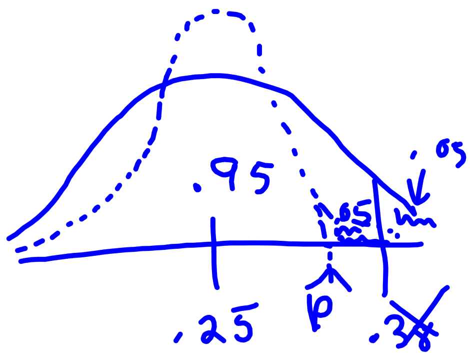
$$\sigma_{\hat{p}} = \sqrt{\frac{.333(1-.333)}{100}} = .0471$$

$$z = \frac{.32 - .333}{.0471} = -.25$$

$$P(z > -.25) = .5987 \approx .60$$

.05	.60 power
.95	.40
1	1

1

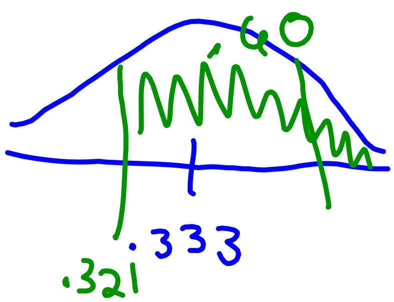


$\uparrow n$ $\quad n = 100$

$$\sigma_{\hat{p}} = \sqrt{\frac{.25(1-.25)}{100}}$$

$$= .0433$$

$$1.645 = \frac{\hat{p} - .25}{.0433} \qquad \hat{p} = .321$$



$$\sigma_{\hat{p}} = \sqrt{\frac{.333(1-.333)}{100}} = .0471$$

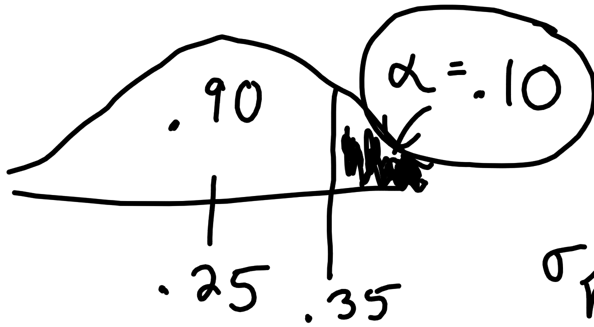
.05	.60
.95	.40

$$z = \frac{.321 - .333}{.0471} = -.25$$

$$P(z > -.25) = .5987$$

$$\approx .60$$

2)



$\sigma_{\hat{p}} = .0791$

invnorm(.90) = 1.28

$$1.28 = \frac{\hat{p} - .25}{.0791}$$

$$\hat{p} = .35$$

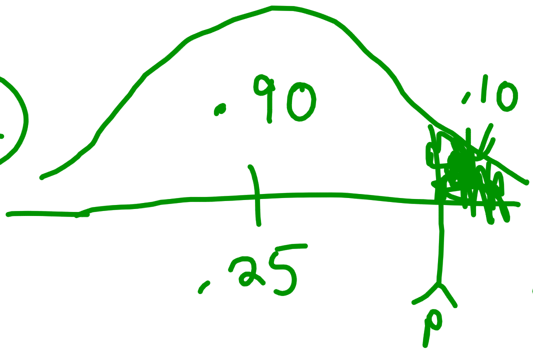
$\sigma_{\hat{p}} = .086$

$$z = \frac{.35 - .333}{.086} = .20$$

.10	.42
.90	.58
1	1

$P(z > .20) = .42$

2



$n = 30$

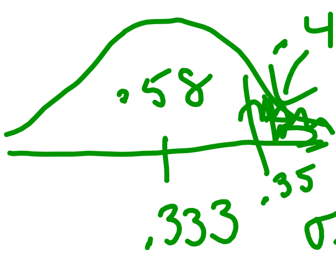
$\alpha = .10$

$\sigma_{\hat{p}} = .0791$

$\text{invnorm}(.90) = 1.28$

$$1.28 = \frac{\hat{p} - .25}{.0791}$$

$$\hat{p} = .35$$



$\sigma_{\hat{p}} = .0860$

$$z = \frac{.35 - .333}{.086} = .20$$

$$P(z > .20) = .4207$$

.10	.42
.90	.58